1.

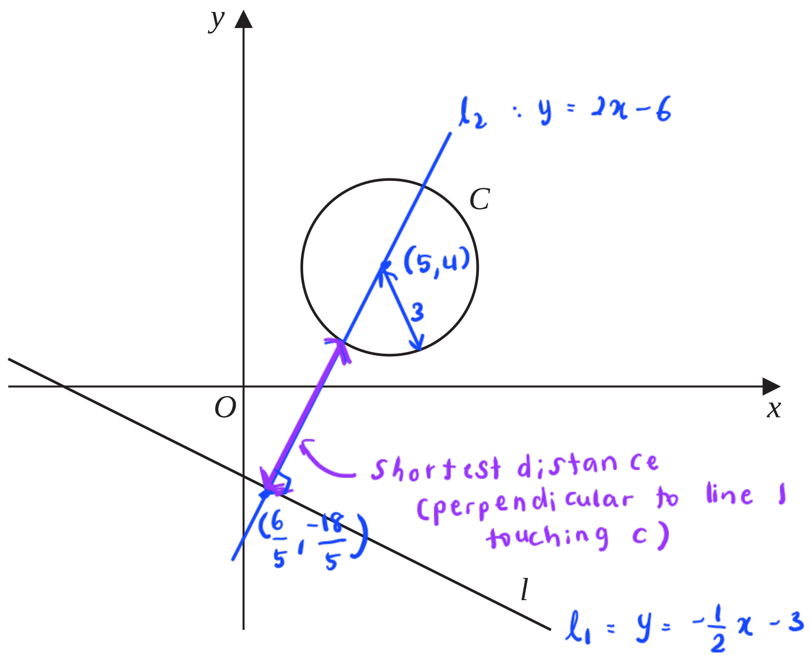


Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the radius of C.

(3)

(b) Find the shortest distance between C and l.

(5)

$(x-a)^2 + (y-b)^2 = r^2$

a) (i) $x^2 + y^2 - 10x - 8y + 32 = 0$

$(x-5)^2 - 5^2 + (y-4)^2 - 4^2 + 32 = 0$

$(x-5)^2 + (y-4)^2 = 9$

$(x-5)^2 + (y-4)^2$ (1) $= 3^2$

$\therefore$ centre $(5,4)$ (1)

(ii) radius $= 3$ (1)

b) $l_1 = 2y + x + 6 = 0$

$2y = -x - 6$

$y = -\frac{1}{2}x - 3$ $\therefore$ gradient of l_1 is $-\frac{1}{2}$

$\therefore$ gradient of l_2 is $\frac{-1}{-\frac{1}{2}} = 2$ (1)

Finding equation of line l_2 :

Known $(5,4)$ from centre of circle :

$y - 4 = 2(x-5)$ (1)

$\therefore l_2 : y = 2x - 6$

Finding intersect point of l_1 and l_2 :

$l_1 : y = -\frac{1}{2}x - 3$ — (1)

$l_2 : y = 2x - 6$ — (2)

subs (2) into (1)

$-\frac{1}{2}x - 3 = 2x - 6$

$-x - 6 = 4x - 12$

$5x = 6 \rightarrow \therefore x = \frac{6}{5}$

$y = 2\left(\frac{6}{5}\right) - 6 \longrightarrow y = -\frac{18}{5}$ (1)

Finding distance from l_1 to centre of C :

$$= \sqrt{\left(5 - \frac{6}{5}\right)^2 + \left(4 - \left(-\frac{18}{5}\right)\right)^2} \quad (1)$$

$$= \frac{19\sqrt{5}}{5}$$

Finding distance from l_1 to C :

radius of C

$$\frac{19\sqrt{5}}{5} - 3 \quad (1)$$

$$= 5.50$$

2. The circle C has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

Given that C does not cut or touch the x-axis,

(b) find the range of possible values for k.

(3)

a) $x^2 + y^2 - 6x + 10y + k = 0$

$x^2 - 6x + y^2 + 10y + k = 0$

$(x-3)^2 + (y+5)^2 - 9 - 25 + k = 0$ (1)

centre of C = $(3, -5)$ (1)

b) If circle touch the x-axis, $y = 0$

Hence, $x^2 - 6x + k = 0$.

However, we know that C does not touch the x-axis.

So, $b^2 - 4ac < 0$

$(-6)^2 - 4(1)(k) < 0$

$4k > 36$

$k > 9$. (1)

Radius should be > 0, so :

$(x-3)^2 + (y+5)^2 = \boxed{9 + 25 - k}$ radius

$9 + 25 - k > 0$ (1)

$k < 34$

Combine the two solutions :

$\therefore \ 9 < k < 34$ (1)

3. The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C,

(b) find the possible values of k, giving your answers as simplified surds.

(5)

a) $C: x^2+y^2-10x+4y+11=0$ complete the square on x and y

$(x-5)^2-25+(y+2)^2-4+11=0$ ①

$(x-5)^2+(y+2)^2=18$ ①

centre: $(5,-2)$ ① radius: $\sqrt{18} = 3\sqrt{2}$ ① simplified surd

b) $L: y = 3x+k$

L is tangent to C, so they have only 1 point of intersection.
Method: sub $y = 3x+k$ into C to form a quadratic in x, then set the discriminant equal to 0.

sub L into C: $x^2+(3x+k)^2-10x+4(3x+k)+11=0$ ①

$x^2+9x^2+6xk+k^2-10x+12x+4k+11=0$

$10x^2+(6k+2)x+(k^2+4k+11)=0$ ①

$$10x^2 + (6k+2)x + (k^2 + 4k + 11) = 0$$

This quadratic has only 1 solution, so discriminant = 0.

"$b^2 - 4ac$" = 0

$$(6k+2)^2 - 4 \times 10 \times (k^2 + 4k + 11) = 0 \quad (1)$$

$$36k^2 + 24k + 4 - 40k^2 - 160k - 440 = 0$$

$$-4k^2 - 136k - 436 = 0$$

$$4k^2 + 136k + 436 = 0 \quad (1)$$

using calculator

$$k = -17 \pm 6\sqrt{5} \quad (1)$$

4. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

(a) (i) $x^2 + y^2 - 10x + 16y = 80$

'complete the square' on x and y terms.

$(x-5)^2 + (y+8)^2 - 5^2 - 8^2 = 80$

$(x-5)^2 + (y+8)^2 = 169$ 1

$x^2 + bx + c \downarrow (x + \frac{b}{2})^2 + c - (\frac{b}{2})^2$

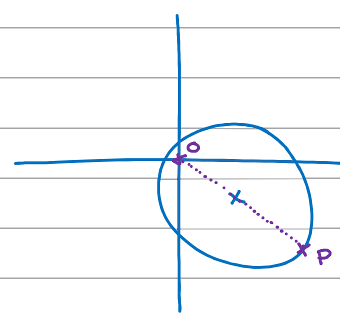
$\therefore$ centre $= (5, -8)$ 1

A circle with centre (a, b) and radius r has equation:
$(x-a)^2 + (y-b)^2 = r^2$

(ii) radius $= \sqrt{169}$
$= 13$ 1

(b) furthest point will be origin → centre + radius.

length $= \underbrace{\sqrt{5^2 + (-8)^2}}_{\text{origin to centre}} + \underbrace{13}_{\text{radius}}$ 1



$= \sqrt{89} + 13$ 1

"exact" so leave in this form.

5. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

(a) Find in terms of k,

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

(b) Find the range of possible values of k.

(6)

a) $x^2 + y^2 + 6kx - 2ky + 7 = 0$

$(x+3k)^2 - 9k^2 + (y-k)^2 - k^2 + 7 = 0$

$(x+3k)^2 + (y-k)^2 = 10k^2 - 7$ 1

(i) centre $(-3k, k)$ 1

(ii) radius $\sqrt{10k^2-7}$ 1

b) sub $y = 2x-1$ into C, then use $b^2 - 4ac > 0$

$x^2 + (2x-1)^2 + 6kx - 2k(2x-1) + 7 = 0$

$x^2 + 4x^2 - 4x + 1 + 6kx - 4kx + 2k + 7 = 0$ 1

$5x^2 + (2k-4)x + (2k+8) = 0$ 1

two distinct solutions, so "$b^2 - 4ac$" > 0

$(2k-4)^2 - 4(5)(2k+8) > 0$ 1

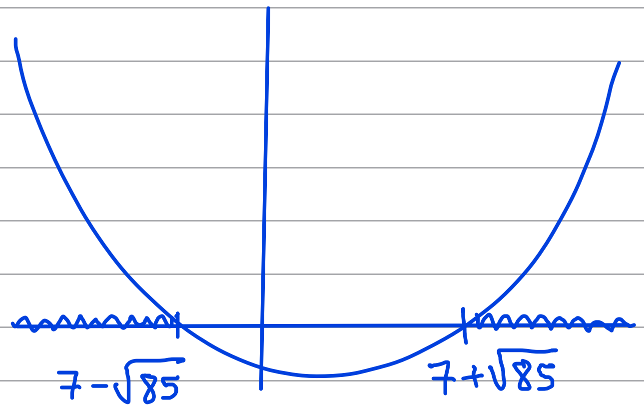
$4k^2 - 16k + 16 - 40k - 160 > 0$

$4k^2 - 56k - 144 > 0$

$k^2 - 14k - 36 > 0$

critical values are solutions to $k^2 - 14k - 36 = 0$

$k = 7 \pm \sqrt{85}$ (1)



~~~ = want this region

$k < 7 - \sqrt{85}$ or $k > 7 + \sqrt{85}$
(1) (1)
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